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ENERGY-MINIMIZING STRAINS IN MARTENSITIC MICROSTRUCTURES

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Abstract : this communication is concerned with the theoretical prediction of the energy-minimizing (or stress-free) strains that can be realized by martensitic microstructures. Polyconvexification and related notions are used to derive some upper bounds (in the sense of inclusion) on the set of energy-minimizing strains. Lower bounds are obtained from lamination techniques. Three-, four-, and twelve-well problems are considered. In particular, the structure of the set of energy-minimizing strains in cubic to monoclinic transformations is investigated in detail.

Keywords : phase transformation, microstructures, energy-minimization, lamination, nonlinear bounds

Some metallic alloys exhibit a solid/solid phase transformation between different crystallographic structures, known as austenite (stable at high temperature) and martensite (stable at low temperature) [1]. That phase transformation can be triggered both by thermal and mechanical loading. In terms of crystallographic structure, the austenite has a higher symmetry than the martensite. Therefore, the martensite actually exists in the form of several variants, corresponding to different orientations of the martensitic lattice with respect to the austenitic lattice. Accordingly, to each martensitic variant is attached a transformation strain, describing the deformation between the crystallographic structures of the austenite and the martensite. The number of martensitic variants as well as the corresponding transformation strains depend on the alloy considered, through the structure of the austenite and martensite lattices. Some common examples include the cubic to tetragonal transformation (MnCu, MnNi), the cubic to orthorombic transformation (β_1'CuAlNi) and the cubic to monoclinic transformations (NiTi, γ_1'CuAlNi), corresponding respectively to 3, 6 and 12 martensitic variants.

This communication is concerned with the theoretical prediction of the set of strains that minimize the effective (or macroscopic) energy. Those strains, classically refered to as recoverable strains, play a central role in shape memory effect displayed by alloys such as NiTi or CuAlNi [3]. The macroscopic energy is defined as the quasiconvexification (or relaxation) of a multi-well energy function that models the behaviour of the material at a microscopic level. The relaxation procedure essentially consists in finding the austenite/martensite microstructures which minimize the global energy. Closed-form solutions have been obtained only for two phases in the geometrically nonlinear setting [2], and up to three phases in the geometrically linear setting [4, 8].

This communication aims at complementing existing results on that problem, essentially by deriving bounds on the set of energy-minimizing strains [5, 6, 7]. Upper bounds are obtained using distinctive properties of Young measures [2]. Lower bounds are constructed using lamination techniques. Both the geometrically nonlinear setting (finite strains) and the geometrically linear setting (infinitesimal strains) are covered, the latter being less accurate but significantly more tractable.

In the geometrically nonlinear setting, analytical expressions of both lower and upper bounds are derived for a general three-well problem that encompasses the cubic to tetragonal transformation as a special case. In the geometrically linear setting, the twelve-well problems corresponding to cubic to monoclinic-I and cubic to monoclinic-II transformations are investigated. The structure of the sets of energy-minimizing strains is studied in detail. For the twelve-well problems, that investigation is notably supported by considering four-well restrictions, for which three-dimensional representations of the bounding sets can be obtained.

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